Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number ☐☐☐☐☐ Candidate Number ☐☐☐☐

Pearson Edexcel Level 3 GCE

Wednesday 14 June 2023

Afternoon (Time: 1 hour 30 minutes)

Paper reference **9FM0/3C**

Further Mathematics

Advanced

PAPER 3C: Further Mechanics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

1. A particle P of $\overset{m}{\text{mass 2 kg}}$ is moving with velocity $\overset{u}{(-4\mathbf{i} + 3\mathbf{j})}$ m s^{-1} when it receives an impulse $\underset{I}{(-6\mathbf{i} + 42\mathbf{j})}$ N s.

(a) Find the speed of P immediately after receiving the impulse.

(4)

The angle through which the direction of motion of P has been deflected by the impulse is $\alpha°$

(b) Find the value of α

(2)

(a) Impulse is the change in momentum

Formula for change in momentum:

$$I = \Delta\text{momentum} = \overset{\text{mass}}{m}v_{\text{final}} - m\underset{\text{velocity}}{v_{\text{initial}}}$$

Substitute into $I = m(v - u)$:

$$\begin{pmatrix} -6 \\ 42 \end{pmatrix} = 2\left[\begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} -4 \\ 3 \end{pmatrix}\right] \quad \text{M1A1}$$

Set up equations for i and j components separately:

$$-6 = 2(a-(-4)) \rightarrow -6 = 2a+8 \quad a = -7$$

$$42 = 2(b-3) \longrightarrow 42 = 2b-6 \quad b = 24$$

$\therefore$ velocity after: $(-7i + 24j)\text{ms}^{-1}$

Use Pythagoras' theorem to get speed:

$$\sqrt{(-7)^2 + (24)^2}^{\,2} \quad \text{M1}$$

$$= \sqrt{49 + 576}$$

$$= \sqrt{625}$$

$$= 25\text{ms}^{-1} \text{ speed} \quad \text{A1}$$

(b) Let's use the scalar product formula: dot product: $\begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} c \\ d \end{pmatrix} = ac + bd$

$$\cos\theta = \frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a}| \times |\underset{\sim}{b}|}$$

Substitute:

$$\cos\theta = \frac{\begin{pmatrix} -7 \\ 24 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ 3 \end{pmatrix}}{(25) \times \sqrt{(4)^2 + (3)^2}} \quad \text{M1}$$

$$\cos\theta = \frac{28 + 72}{25 \times 5}$$

$$\theta = \cos^{-1}\left(\frac{100}{125}\right)$$

$$\theta = 37° \text{ to 2sf} \quad \text{A1}$$

Question 1 continued

(Total for Question 1 is 6 marks)

2. A car of mass 1000 kg moves in a straight line along a horizontal road at a constant speed U m s^{-1}. The resistance to the motion of the car is a constant force of magnitude 400 N.

The engine of the car is working at a constant rate of 16 kW.

(a) Find the value of U.

(3)

The car now pulls a trailer of mass 600 kg in a straight line along the road using a tow rope which is parallel to the direction of motion. The resistance to the motion of the car is again a constant force of magnitude 400 N. The resistance to the motion of the trailer is a constant force of magnitude 300 N.

The engine of the car is working at a constant rate of 16 kW.

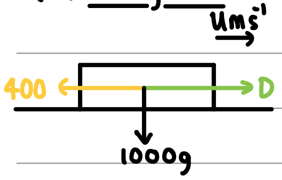
The tow rope is modelled as being light and inextensible.

Using the model,

(b) find the tension in the tow rope at the instant when the speed of the car is $\frac{20}{3}$ m s^{-1}

(5)

(a) Diagram



Since the speed is constant, use $\Sigma F_x = 0$

$D = 400$ M1

To get u we will use Power.

Formula for Power:

Power (W) ⟋ $P = Dv$

Driving force (N) ⟋ ⟍ velocity (ms⁻¹)

Substitute:

$P = 16\text{kW} \xrightarrow{\times 1000} 16\,000\text{W}$

$v = u$

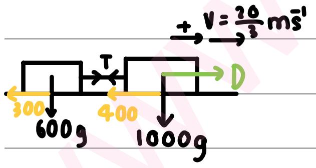
$D = 400$

$160\cancel{0}0 = 40\cancel{0}u$ M1

$40 = u$

$\boxed{u = 40 \text{ms}^{-1}}$ A1

(b) Diagram



To get D we will use Power.

Formula for Power:

Power (W) ⟋ $P = Dv$

Driving force (N) ⟋ ⟍ velocity (ms⁻¹)

$P = 16\text{kW} \xrightarrow{\times 1000} 16\,000\text{W}$

$D = D$

$v = \frac{20}{3}\text{ms}^{-1}$

Substitute: $D = \dfrac{16\,000}{\frac{20}{3}} = 2400\text{N}$ M1

We use $\Sigma F_x = ma$ on the whole system: M1

$F - 700 = 1600a$ $a = \frac{17}{16}\text{ms}^{-2}$ A1

Use $\Sigma F_x = ma$ only on the trailer to get T: M1

$T - 300 = 600a$ $T = 600 \times \frac{17}{16} + 300 = 937.5\text{N}$

$\therefore$ $\boxed{938\text{ N to 3sf}}$ A1

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 8 marks)

3. A particle P of mass $2m$ is moving in a straight line with speed $3u$ on a smooth horizontal plane. It collides directly with a particle Q of mass m that is moving on the plane with speed $2u$ in the opposite direction to P.

The coefficient of restitution between P and Q is e, where $e > \frac{4}{5}$

(a) Show that the speed of Q immediately after the collision is $\dfrac{(4+10e)u}{3}$

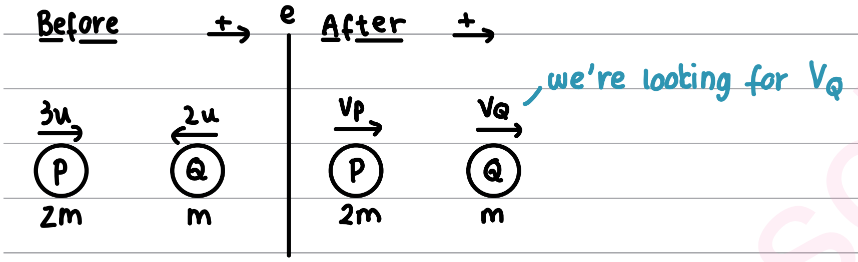
(6)

After the collision Q hits a smooth fixed vertical wall that is perpendicular to the direction of motion of Q. The coefficient of restitution between Q and the wall is f.

(b) Find, **in terms of e**, the set of values of f for which there will be a second collision between P and Q.

(4)

(a) Diagram



We can use the conservation of linear momentum to get an equation

conservation of linear momentum means: the total momentum before the collision is the same as the total momentum after.

Formula: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$ M1

initial velocity final velocity

Substitute: $2m(3u) + m(-2u) = 2m(V_P) + mV_Q$

$6\cancel{m}u - 2\cancel{m}u = 2\cancel{m}V_P + \cancel{m}V_Q$

$4u = 2V_P + V_Q$ Eq1 A1

We can use Newton's Law of Restitution to get an equation.

Newton's Law of Restitution states that: when two objects collide, their speeds after the collision depend on ① speeds before the collision and ② the material from which they're made.

Formula: $e(u_A - u_B) = v_B - v_A$ M1

coefficient of restitution initial speed final speed

Substitute: $e(3u - -2u) = V_Q - V_P$

$5eu = V_Q - V_P$ Eq2 A1

Solve simultaneously Eq1 and Eq2:

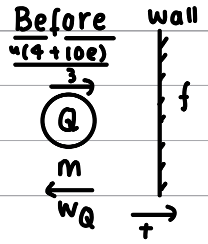
$V_P = V_Q - 5eu \xrightarrow{\text{substitute}} 4u = 2(V_Q - 5eu) + V_Q$ dM1

$4u + 10eu = 3V_Q$

A1 $\boxed{\frac{u(4+10e)}{3} = V_Q}$ hence shown

Question 3 continued

(b) We need to get the speed of Q after the collision with the wall.



$w_Q = \frac{fu(4+10e)}{3}$ B1

We also need the speed V_P :

substitute

$4u = 2v_P + V_Q \leftarrow \frac{u(4+10e)}{3} = V_Q$

$4u = 2v_p + \frac{4u}{3} + \frac{10eu}{3}$

$\frac{8}{3}u - \frac{10eu}{3} = 2v_p$

$\frac{u(8-10e)}{6} = v_p$ → this changed direction as $e > \frac{4}{5}$

$V_P = \frac{u(4-5e)}{3}$ M1

For P and Q to collide again, the speed of Q must be larger than the speed of P. Since w_Q is moving in the negative direction, $w_Q = -\frac{fu(4+10e)}{3}$. Both are moving in the negative direction so we want w_Q to be more negative (and ∴ "smaller" than V_P), so that its magnitude and hence speed is larger.

$w_Q < V_P$

$-\frac{f\cancel{u}(4+10e)}{\cancel{3}} < \frac{\cancel{u}(4-5e)}{\cancel{3}}$ M1 cancel u's and 3's

x-1 $-f < \frac{4-5e}{4+10e}$ x-1 — we're multiplying by a -ive number ∴ flip the inequality sign

$f > \frac{5e-4}{4+10e}$

Since it's a coefficient of restitution, f must be smaller than 1.

$\therefore \frac{5e-4}{4+10e} < f \leq 1$ range for f A1

Question 3 continued

Question 3 continued

(Total for Question 3 is 10 marks)

4. A light elastic string has natural length $2a$ and modulus of elasticity $4mg$.
One end of the elastic string is attached to a fixed point O. A particle P of mass m is attached to the other end of the elastic string.
The particle P hangs freely in equilibrium at the point E, which is vertically below O

(a) Find the length OE.

(4)

Particle P is now pulled vertically downwards to the point A, where $OA = 4a$, and released from rest. The resistance to the motion of P is a constant force of magnitude $\frac{1}{4}mg$.

(b) Find, in terms of a and g, the speed of P after it has moved a distance a.

(7)

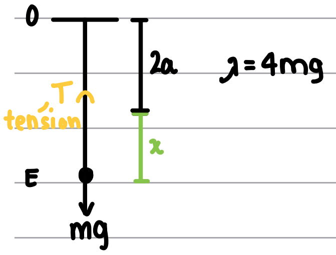
Particle P is now held at O
Particle P is released from rest and reaches its maximum speed at the point B.

The resistance to the motion of P is again a constant force of magnitude $\frac{1}{4}mg$.

(c) Find the distance OB.

(4)

(a) <u>Diagram</u>



Since it's in equilibrium, at E $\Sigma F_y = 0$ applies.

$$T = mg$$

To get T use formula

$$T = \frac{\lambda x}{l}$$

modulus of elasticity
extension
natural length

Substitute:

$$T = \frac{4mgx}{2a}$$

Sub. this back into $\Sigma F_y = 0$

$$\frac{4\cancel{m}gx}{2a} = \cancel{m}g$$

$$4x = 2a$$

$$x = \frac{a}{2}$$

Length OE is natural length + extension.

$$\therefore \boxed{OE = \frac{5}{2}a}$$

Question 4 continued

(a) Use the work-energy principle

★Work-Energy Principle: an increase of KE/GPE/EPE is caused by an equal amount of positive work done on the body(e.g. engine) and a decrease of KE/GPE/EPE is caused by an equal amount of negative work done on the body(e.g. friction).

★ Remember the work-energy formulae:

final grav. potential

Either: $\text{WD by force} + KE_i + GPE_i + EPE_i = KE_F + GPE_F + EPE_F + \text{WD against friction}$

work done, initial kinetic, initial grav. potential, initial elastic potential, final kinetic, final elastic potential, work lost to friction

final grav. potential

OR: $\text{WD by force} + KE_i + GPE_i + EPE_i - \text{WD by friction} = KE_F + GPE_F + EPE_F$

work done, initial kinetic, initial grav. potential, initial elastic potential, we subtract this since it leaves the system as heat!, final kinetic, final elastic potential

★Formulae for KE, GPE and EPE:

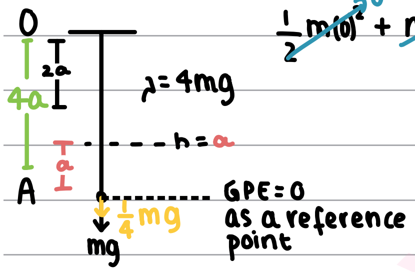
$KE = \frac{1}{2}mv^2$ velocity, mass

$GPE = mgh$ change in height, mass, $g = 9.8ms^{-2}$

modulus of elasticity

$EPE = \frac{\lambda x^2}{2l}$ extension of string/spring, natural length of the string/spring

Diagram



Substitute:

extension, resistive force

$$\cancel{\frac{1}{2}m(0)^2}^{0} + \cancel{mg(0)}^{0} + \frac{4mg(2a)^2}{2(2a)} - a\left(\frac{1}{4}mg\right) = \frac{1}{2}mv^2 + mg(a) + \frac{4mg(a)^2}{2(2a)}$$

$$\frac{4\cancel{m}g(4a^{\cancel{2}})}{\cancel{4a}} - \frac{1}{4}\cancel{m}ga = \frac{\cancel{m}v^2}{2} + \cancel{m}ga + \frac{\cancel{4}\cancel{m}ga^{\cancel{2}}}{\cancel{4a}}$$

$$4ga - \frac{ga}{4} = \frac{v^2}{2} + ga + ga$$

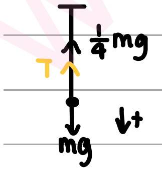
$$2\left(2ga - \frac{ga}{4}\right) = v^2$$

$$4ga - \frac{ga}{2} = v^2$$

$$\boxed{\sqrt{\frac{7}{2}ga} = v}$$ speed after distance a

(c) Maximum speed will be reached when the particle is in equilibrium vertically, ∴ when $\Sigma F_y = 0$. Let extension be e. At B: (↓+)

Diagram



$$mg - T - \frac{1}{4}mg = 0$$

T at this point: $T = \frac{4mge}{2a}$

$$\cancel{mg} = \frac{4\cancel{mg}e}{2a} + \frac{1}{4}\cancel{mg}$$

$$1 = \frac{4e}{2a} + \frac{1}{4}$$

$$\frac{3}{4} = \frac{4e}{2a} \qquad e = \frac{3}{8}a$$

Length OE is natural length + extension ∴ $\boxed{OB = \frac{19}{8}a}$

Question 4 continued

Question 4 continued

(Total for Question 4 is 15 marks)

5.

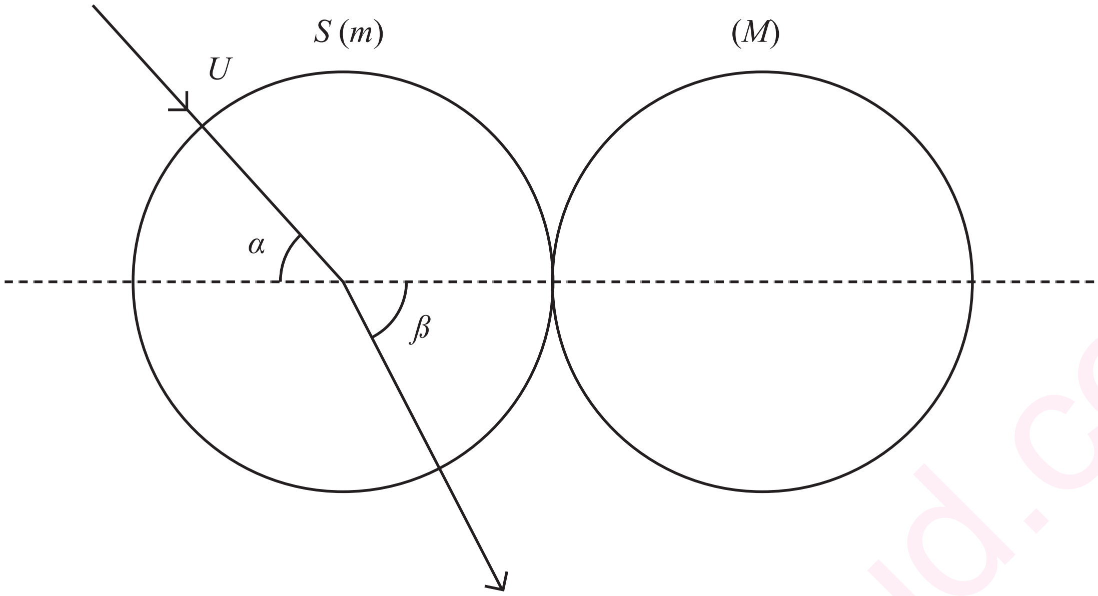


Figure 1

A smooth uniform sphere S of mass m is moving with speed U on a smooth horizontal plane. The sphere S collides obliquely with another uniform sphere of mass M which is at rest on the plane. The two spheres have the same radius.

Immediately before the collision the direction of motion of S makes an angle α, where $0 < \alpha < 90^\circ$, with the line joining the centres of the spheres.

Immediately after the collision the direction of motion of S makes an angle β with the line joining the centres of the spheres, as shown in Figure 1.

The coefficient of restitution between the spheres is e.

(a) Show that $\tan\beta = \dfrac{(m + M)\tan\alpha}{(m - eM)}$

(8)

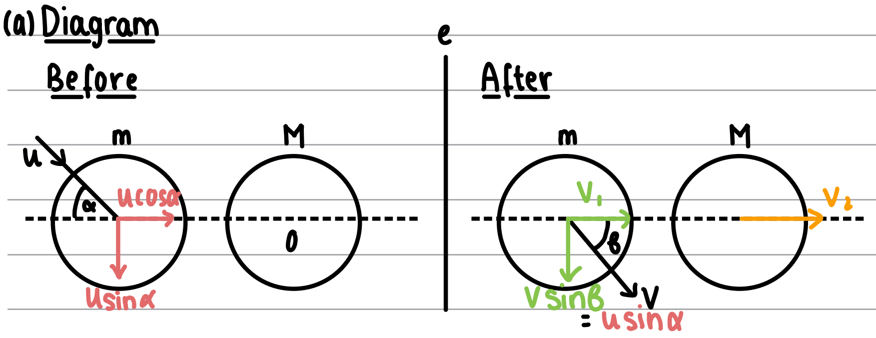
Given that $m = eM$,

(b) show that the directions of motion of the two spheres immediately after the collision are perpendicular.

(2)

Question 5 continued

(a) Diagram



I. Parallel to the line of centers (velocity changes)

We can use the conservation of linear momentum to get this.

conservation of linear momentum means: the total momentum before the collision is the same as the total momentum after.

Formula: $m_A \underbrace{u_A}_{\text{initial velocity}} + m_B u_B = m_A \underbrace{v_A}_{\text{final velocity}} + m_B v_B$

Substitute: $m(u\cos\alpha) + \overset{0}{\cancel{M(0)}} = m(v_1) + M(v_2)$

$$mu\cos\alpha = mv_1 + Mv_2 \quad \text{Eq1}$$

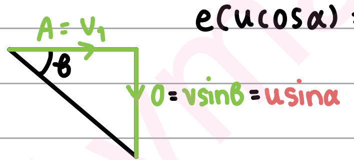
We can use Newton's Law of Restitution to get an equation.

Newton's Law of Restitution states that: when two objects collide, their speeds after the collision depend on ① speeds before the collision and ② the material from which they're made.

Formula: $\underbrace{e}_{\text{coefficient of restitution}}(\underbrace{u_A}_{\text{initial speed}} - u_B) = v_B - \underbrace{v_A}_{\text{final speed}}$

$$e(u\cos\alpha) = v_2 - v_1 \quad \text{Eq2}$$

Visualize tanβ:



$\tan\beta = \frac{O}{A} = \frac{u\sin\alpha}{v_1}$

Let's get v_1 by solving Eq1 and Eq2 simultaneously

$$mu\cos\alpha = mv_1 + Mv_2$$

$$e(u\cos\alpha) = v_2 - v_1 \rightarrow v_2 = eu\cos\alpha + v_1$$

$$mu\cos\alpha = mv_1 + M(eu\cos\alpha + v_1)$$

$$mu\cos\alpha - Meu\cos\alpha = v_1(m+M)$$

$$\frac{u\cos\alpha(m - Me)}{(m+M)} = v_1$$

Substitute into tanβ: $\tan\beta = \frac{\cancel{u}\sin\alpha}{\frac{\cancel{u}\cos\alpha(m-Me)}{m+M}}$ $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$

$\downarrow$

$$\boxed{\tan\beta = \frac{(m+M)\tan\alpha}{(m-Me)}}$$ hence shown

Question 5 continued

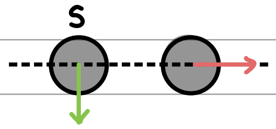
(b) If $m = eM$:

$$\tan\beta = \frac{(m+M)\tan\alpha}{\underbrace{(eM - eM)}_{=0}}$$

As the denominator ends up equal to 0,

$\tan\beta = \infty$, $\therefore$ $\beta = 90°$ (from the asymptote of the tan graph)

So after the collision, S moves perpendicular to the line of centers. The other sphere moves parallel to the line of centers.



Hence the spheres move perpendicularly to eachother.

Question 5 continued

(Total for Question 5 is 10 marks)

6. A particle P of mass m is falling vertically when it strikes a fixed smooth inclined plane. The plane is inclined to the horizontal at an angle α, where $0 < \alpha \leqslant 45°$

At the instant immediately before the impact, the speed of P is u.

At the instant immediately after the impact, P is moving horizontally with speed v.

(a) Show that the magnitude of the impulse exerted on the plane by P is $mu\sec\alpha$

(5)

The coefficient of restitution between P and the plane is e, where $e > 0$

(b) Show that $v^2 = u^2(\sin^2\alpha + e^2\cos^2\alpha)$

(3)

(c) Show that the kinetic energy lost by P in the impact is

$$\frac{1}{2}mu^2(1 - e^2)\cos^2\alpha$$

(2)

(d) Hence find, in terms of m, u and e **only**, the kinetic energy lost by P in the impact.

(2)

(a) Method 1

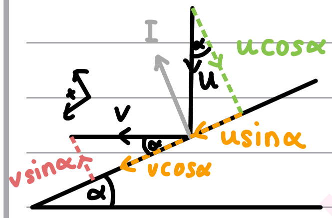
We can use the conservation of linear momentum along the plane.

conservation of linear momentum means: the total momentum before the collision is the same as the total momentum after.

Formula: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

initial velocity   final velocity

Diagram



Substitute:

$$\cancel{m}(u\sin\alpha) = \cancel{m}(v\cos\alpha) \quad \text{M1A1}$$

$$u\sin\alpha = v\cos\alpha$$

$$\boxed{v = \frac{u\sin\alpha}{\cos\alpha}}$$

Perpendicular to the plane we will use the impulse-momentum principle.

Impulse is the change in momentum

Formula for change in momentum:

$$I = \Delta\text{momentum} = m v_{\text{final}} - m v_{\text{initial}}$$

mass

initial velocity

Substitute:

$$I = mv - mu = m(v - u) \quad \text{M1}$$

$$= m(v\sin\alpha - (-u\cos\alpha)) \quad \text{A1}$$

$\frac{u\sin\alpha}{\cos\alpha}$ from above

subst. v.

$$= m(v\sin\alpha + u\cos\alpha)$$

$$= m\left(\frac{u\sin^2\alpha}{\cos\alpha} + u\cos\alpha\right)$$

$$u\cos\alpha \div \frac{u}{\cos\alpha} = \cancel{u}\cos\alpha \times \frac{\cos\alpha}{\cancel{u}} = \cos^2\alpha$$

factorize $= \frac{mu}{\cos\alpha}(\sin^2\alpha + \cos^2\alpha)$ trig. identity $\sin^2 a + \cos^2 a = 1$

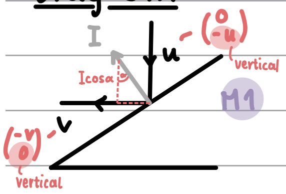
$$\frac{1}{\cos\alpha} = \sec\alpha = \frac{mu}{\cos\alpha} = \boxed{mu\sec\alpha}$$

hence shown A1

Question 6 continued

Method 2 We will use the Impulse-Momentum principle vertically M1

Diagram Impulse is the change in momentum



M1

Formula for change in momentum:

$$I = \Delta\text{momentum} = \overset{\text{mass}}{m}v_{\text{final}} - mv_{\text{initial}}$$

velocity

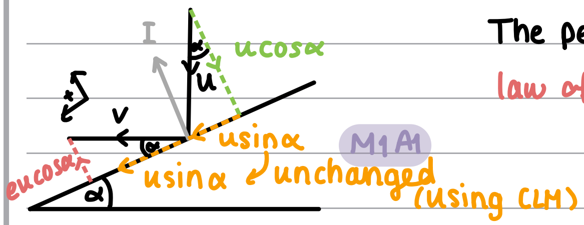
$$I\cos\alpha = m(0 - -u)$$ A1 A1

speed vertically is 0 since the particle moves horizontally

$$I\cos\alpha = mu$$

$$I = \frac{mu}{\cos\alpha} = mu\sec\alpha$$ hence shown A1

Method 3



M1 A1

The perpendicular component of v is found using Newton's law of Restitution:

$$e \times u\cos\alpha = eu\cos\alpha$$

From the diagram we see that:

$$\tan\alpha = \frac{e\cancel{u}\cos\alpha}{\cancel{u}\sin\alpha} = e\frac{\cos\alpha}{\sin\alpha}$$

$$\therefore \quad e = \tan\alpha \times \frac{\sin\alpha}{\cos\alpha} = \boxed{\tan^2\alpha = e}$$ M1

Use the Impulse - Momentum principle perpendicularly:

$$I = mv - mu = m(v - u)$$

$$I = m(eu\cos\alpha - (-u\cos\alpha))$$

$$I = m(\tan^2\alpha\, u\cos\alpha + u\cos\alpha)$$ A1

$$\tan^2\alpha = \frac{\sin^2\alpha}{\cos^2\alpha}$$

$$I = mu\left(\frac{\sin^2\alpha}{\cos^2\alpha} \times \cancel{\cos\alpha} + \cos\alpha\right)$$ factor out u

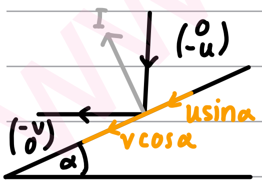
$$u\cos\alpha \div \frac{u}{\cos\alpha} = \cancel{u}\cos\alpha \times \frac{\cos\alpha}{\cancel{u}} = \cos^2\alpha$$

$$I = mu\left(\frac{\sin^2\alpha}{\cos\alpha} + \cos\alpha\right)$$

factorize $= \frac{mu}{\cos\alpha}(\sin^2\alpha + \cos^2\alpha)$ trig. identity $\sin^2 a + \cos^2 a = 1$

$$\frac{1}{\cos\alpha} = \sec\alpha = \frac{mu}{\cos\alpha} = \boxed{mu\sec\alpha}$$ hence shown A1

Method 4 - Vectors Use CLM along the plane: M1



$$\cancel{m}u\sin\alpha = \cancel{m}v\cos\alpha$$

A1 $v = u \times \frac{\sin\alpha}{\cos\alpha} = u\tan\alpha$ (using trig. $\tan\theta = \frac{\sin\theta}{\cos\theta}$)

Impulse is the change in momentum

Formula for change in momentum:

$$I = \Delta\text{momentum} = \overset{\text{mass}}{m}v_{\text{final}} - mv_{\text{initial}}$$

velocity

$$I = m\left(\begin{pmatrix}-v\\0\end{pmatrix} - \begin{pmatrix}0\\-u\end{pmatrix}\right) = m\begin{pmatrix}-v\\u\end{pmatrix}$$ M1

Question 6 continued

Apply Pythagoras' Theorem to get the magnitude of the impulse:

$|I| = m\sqrt{v^2+u^2}$

$= m\sqrt{(u\tan\alpha)^2+u^2}$ $v = u\tan\alpha$ (see above)

A1 $= m\sqrt{u^2\tan^2\alpha+u^2}$ use trig. identity

$= m\sqrt{u^2}\sqrt{\tan^2\alpha+1}$ $1+\tan^2\alpha = \sec^2\alpha$

$= mu\sqrt{\sec^2\alpha} = \boxed{mu\sec\alpha = I}$ hence shown A1

(b) We can use Newton's Law of Restitution to get an equation perpendicularly

Newton's Law of Restitution states that: when two objects collide, their speeds after the collision depend on ① speeds before the collision and ② the material from which they're made.

Formula: $e(u_A - u_B) = v_B - v_A$

coefficient of restitution initial speed final speed

Substitute: M1 $eu\cos\alpha = v\sin\alpha$

Along the plane → $u\sin\alpha = v\cos\alpha$

Square and add: M1

$e^2u^2\cos^2\alpha + u^2\sin^2\alpha = v^2\sin^2\alpha + v^2\cos^2\alpha$

$u^2(e^2\cos^2\alpha + \sin^2\alpha) = v^2(\sin^2\alpha+\cos^2\alpha)$ trig identity: $\sin^2\alpha+\cos^2\alpha = 1$

$\therefore$ $\boxed{u^2(e^2\cos^2\alpha+\sin^2\alpha) = v^2}$ shown.

A1

(c) To get KE lost: $\Delta KE = KE_I - KE_F$

Formula for Kinetic Energy:

$KE = \frac{1}{2}mv^2$

mass velocity

Substitute: $\Delta KE = \frac{1}{2}mu^2 - \frac{1}{2}mv^2$ found in (b)

$= \frac{1}{2}mu^2 - \frac{1}{2}mu^2(\sin^2\alpha + e^2\cos^2\alpha)$ M1

factorize $= \frac{1}{2}mu^2\left[1 - \sin^2\alpha - e^2\cos^2\alpha\right]$

$\sin^2\alpha + \cos^2\alpha = 1$, $\cos^2\alpha = 1 - \sin^2\alpha$

$= \frac{1}{2}mu^2\left[\cos^2\alpha - e^2\cos^2\alpha\right]$

$= \boxed{\frac{1}{2}mu^2\cos^2\alpha(1-e^2)}$ hence shown A1

(d) We found that $e = \tan^2\alpha$ in (a), Method 3. $\tan^2\alpha = \sec^2\alpha - 1 \rightarrow e = \frac{1}{\cos^2\alpha} - 1$

Substitute: $KE_{lost} = \frac{1}{2}mu^2\frac{1}{e+1}(1-e^2)$ $e+1 = \frac{1}{\cos^2\alpha} \rightarrow \cos^2\alpha = \frac{1}{e+1}$

M1A1

Question 6 continued

(Total for Question 6 is 12 marks)

7.

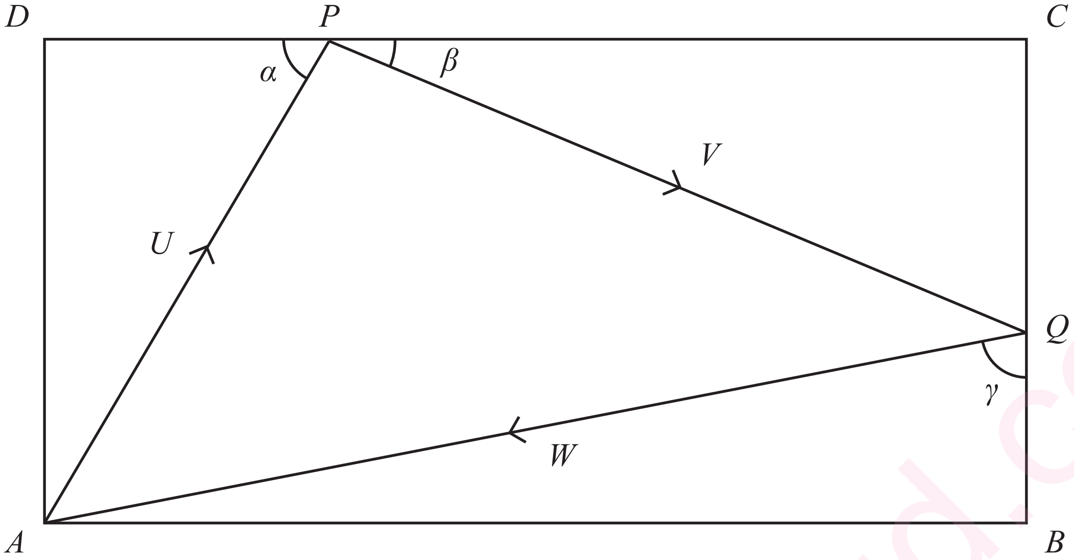


Figure 2

A small smooth snooker ball is projected from the corner *A* of a horizontal rectangular snooker table *ABCD*.

The ball is projected so it first hits the side *DC* at the point *P*, then hits the side *CB* at the point *Q* and then returns to *A*.

Angle *APD* = α, Angle *QPC* = β, Angle *AQB* = γ
The ball moves along *AP* with speed *U*, along *PQ* with speed *V* and along *QA* with speed *W*, as shown in Figure 2.

The coefficient of restitution between the ball and side *DC* is e_1

The coefficient of restitution between the ball and side *CB* is e_2

The ball is modelled as a particle.

Use the model to answer all parts of this question.

(a) Show that $\tan\beta = e_1 \tan\alpha$

(4)

(b) Hence show that $e_1 \tan\alpha = e_2 \cot\gamma$

(3)

(c) By considering (angle *APQ* + angle *AQP*) or otherwise, show that it would be possible for the ball to return to *A* only if $e_2 > e_1$

(6)

If instead $e_1 = e_2$, the ball would **not** return to *A*.

Given that $e_1 = e_2$

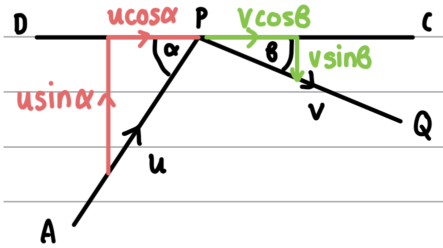
(d) use the result from part (b) to describe the path of the ball after it hits *CB* at *Q*, explaining your answer.

(1)

Question 7 continued

(a) We will consider the first collision:

Diagram



Parallel to the wall the velocity doesn't change:

$$u\cos\alpha = v\cos\beta \quad \text{B1}$$

Perpendicular to the wall we use Newton's Law of Restitution:

$$e_1 u\sin\alpha = v\sin\beta \quad \text{B1}$$

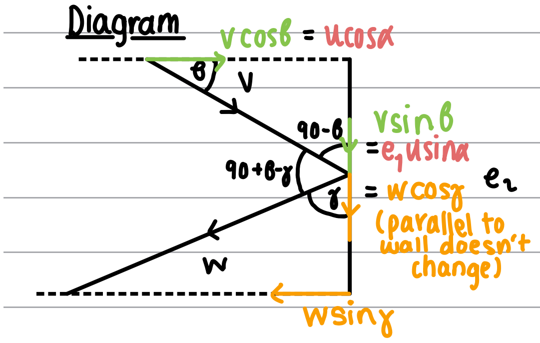
Divide the two equations:

$$\frac{e_1\cancel{u}\sin\alpha}{\cancel{u}\cos\alpha} = \frac{\cancel{v}\sin\beta}{\cancel{v}\cos\beta} \quad \text{cancel } v \text{ and } u \quad \text{M1}$$

$$e_1\tan\alpha = \tan\beta \quad \text{shown} \quad \text{A1}$$

(b) Now we will consider the second collision.

Diagram



To get tanγ we need sinγ and cosγ.

parallel $v\sin\beta = w\cos\gamma$

perpendicular $e_2 v\cos\beta = w\sin\gamma$

$$\tan\gamma = \frac{\cancel{w}\sin\gamma}{\cancel{w}\cos\gamma} = \frac{e_2\cancel{v}\cos\beta}{\cancel{v}\sin\beta} \quad \text{B1}$$

$$\tan\gamma = e_2\cot\beta$$

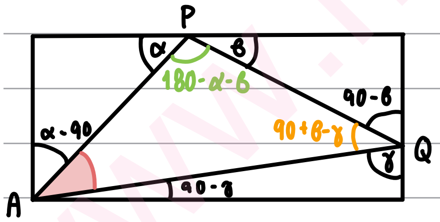
and $\tan\gamma = e_2 \times \dfrac{1}{\tan\beta} = e_2 \times \dfrac{1}{e_1\tan\alpha}$ from (a)

$$\therefore \quad \tan\gamma = e_2 \times \frac{1}{e_1\tan\alpha} \quad \text{M1}$$

$$e_1\tan\alpha = \frac{e_2}{\tan\gamma}$$

$$\therefore \quad e_1\tan\alpha = e_2\cot\gamma \quad \text{shown} \quad \text{A1}$$

(c) Diagram



angle $P\hat{A}Q = 180 - (A\hat{P}Q + A\hat{Q}P)$

$= 180 - (180 - \alpha - \beta + 90 + \beta - \gamma)$

$= \alpha + \gamma - 90$ angle $P\hat{A}Q$ M1

For it to return to A, angle $P\hat{A}Q$ must be larger than 0. M1

$$\alpha + \gamma - 90 > 0$$

A1 $\alpha > 90 - \gamma$

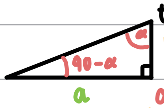
$\tan\alpha > \tan(90-\gamma)$ $= \cot\gamma$ M1

from (b): $\tan\alpha = \dfrac{e_2\cot\gamma}{e_1}$

$$\frac{e_2\cancel{\cot\gamma}}{e_1} > \cancel{\cot\gamma} \quad \text{cancel } \cot\gamma \quad \text{M1}$$

$$\frac{e_2}{e_1} > 1$$

$$e_2 > e_1 \quad \text{hence shown} \quad \text{A1}$$



$\tan\alpha = \frac{O}{A} = \frac{a}{b}$

these are reciprocal!

$\tan(90-\alpha) = \frac{b}{a}$ $\therefore \tan(90-\alpha) = \cot\alpha$

Question 7 continued

(d) From (b) : $\alpha = 90 - \gamma$ $\therefore$ it moves parallel to AP B1

Question 7 continued

Question 7 continued

(Total for Question 7 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS